## Money and Banking

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## Lecture 3 Understanding Interest Rate

- Measuring Interest Rate
- Present value
- Four types of credit market instruments
- simple loan
- fixed-payment loan
- coupon bond
- discount bond
- Yield to maturity
- The distinction between interest rates and return.
- Maturity and the volatility of bond returns: interest-rate risk
- The distinction between real and nominal interest rates


## Present Value

- Different debt instrument has different streams of cash payment (cash flows) with very different timing
- The concept of present value (present discounted value) is based on the common notion that a dollar one year from now is less than one dollar.
- simple loan: a loan where the lender provide the borrower with an amount of funds(principle) that must be paid back at the maturity date with an interest payment.


## Discounting the Future:

$$
P V=\frac{C F}{(1+i)^{n}}
$$

PV: the present value
CF: the amount of cash flow
i: nominal interest rate
n : number of years of the cash flow from now

## Credit Market Instruments

- Four types of credit markets instruments:
- simple loan
- fixed-payment loan(fully amortized loan): a loan that must be repaid by making the same payment every period
- coupon bond: pays the owner a fixed interest payment every year and the face value(par value) at the maturity
- coupon rate: yearly coupon payment/face value
- discounted coupon: coupon sold at a price lower than the face value and paid in face value at maturity
- Yield to maturity: interest rate that equates the present value of cash flow payments received from a debt instrument with its value today
- the yield to maturity of a simple loan equals its interest rate

The same cash flow payment every period throughout

$$
\begin{gathered}
\text { the life of the loan } \\
\text { LV }=\text { loan value } \\
\mathrm{FP}=\text { fixed yearly payment } \\
n=\text { number of years until maturity } \\
\mathrm{LV}=\frac{\mathrm{FP}}{1+i}+\frac{\mathrm{FP}}{(1+i)^{2}}+\frac{\mathrm{FP}}{(1+i)^{3}}+\ldots+\frac{\mathrm{FP}}{(1+i)^{n}}
\end{gathered}
$$

We need financial calculators to solve for the yield to maturity i.

Using the same strategy used for the fixed-payment loan:

$$
\begin{gathered}
\mathrm{P}=\text { price of coupon bond } \\
\mathrm{C}=\text { yearly coupon payment } \\
\mathrm{F}=\text { face value of the bond } \\
n=\text { years to maturity date } \\
\mathrm{P}=\frac{\mathrm{C}}{1+i}+\frac{\mathrm{C}}{(1+i)^{2}}+\frac{\mathrm{C}}{(1+i)^{3}}+\ldots+\frac{\mathrm{C}}{(1+i)^{n}}+\frac{\mathrm{F}}{(1+i)^{n}}
\end{gathered}
$$

[^0]
## Table 1 Yields to Maturity on a 10\%-Coupon-Rate Bond Maturing in Ten Years (Face Value $=\$ 1,000$ )

## Yields to Maturity on a 10\%-Coupon-Rate Bond Maturing in Ten Years (Face Value = \$1,000)

| Price of Bond (\$) | Yield to Maturity (\%) |
| :---: | :---: |
| 1,200 | 7.13 |
| 1,100 | 8.48 |
| 1,000 | 10.00 |
| 900 | 11.75 |
| 800 | 13.81 |

- When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate
- The price of a coupon bond and the yield to maturity are negatively related
- The yield to maturity is greater than the coupon rate when the bond price is below its face value
- A bond with no maturity date that does not repay principal but pays fixed coupon payments forever

$$
\begin{aligned}
& P=C / i_{c} \\
& P_{c}=\text { price of the consol } \\
& C=\text { yearly interest payment } \\
& i_{c}=\text { yield to maturity of the consol }
\end{aligned}
$$

can rewrite above equation as this : $i_{c}=C / P_{c}$

For coupon bonds, this equation gives the current yield, an easy to calculate approximation to the yield to maturity for long term bonds.

For any one year discount bond

$$
i=\frac{\mathrm{F}-\mathrm{P}}{\mathrm{P}}
$$

$\mathrm{F}=$ Face value of the discount bond
$\mathrm{P}=$ current price of the discount bond
The yield to maturity equals the increase in price over the year divided by the initial price.
As with a coupon bond, the yield to maturity is negatively related to the current bond price.

The payments to the owner plus the change in value expressed as a fraction of the purchase price

$$
\mathrm{RET}=\frac{\mathrm{C}}{\mathrm{P}_{t}}+\frac{\mathrm{P}_{t+1}-\mathrm{P}_{t}}{\mathrm{P}_{t}}
$$

RET $=$ return from holding the bond from time $t$ to time $t+1$

$$
\begin{gathered}
\mathrm{P}_{t}=\text { price of bond at time } t \\
\mathrm{P}_{t+1}=\text { price of the bond at time } t+1 \\
\mathrm{C}=\text { coupon payment } \\
\frac{\mathrm{C}}{\mathrm{P}_{t}}=\text { current yield }=i_{c} \\
\frac{\mathrm{P}_{t+1}-\mathrm{P}_{t}}{\mathrm{P}_{t}}=\text { rate of capital gain }=g
\end{gathered}
$$

- The return equals the yield to maturity only if the holding period equals the time to maturity
- A rise in interest rates is associated with a fall in bond prices, resulting in a capital loss if time to maturity is longer than the holding period
- The more distant a bond's maturity, the greater the size of the percentage price change associated with an interest-rate change
- The more distant a bond's maturity, the lower the rate of return the occurs as a result of an increase in the interest rate
- Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise


## Table 2 One-Year Returns on Different-Maturity 10\%-Coupon-Rate Bonds

One-Year Returns on Different-Maturity 10\%-Coupon-Rate Bonds When Interest Rates
Rise from 10\% to 20\%

| (1) | (2) |  | (4) | (5) | (6) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Years to | Initial | (3) | Price | Rate of | Rate of |
| Maturity | Current | Initial | Next | Capital | Return |
| When Bond | Yield | Price | Year* | Gain | $(2+5)$ |
| Is Purchased | (\%) | (\$) | (\$) | (\%) | (\%) |
| 30 | 10 | 1,000 | 503 | -49.7 | -39.7 |
| 20 | 10 | 1,000 | 516 | -48.4 | -38.4 |
| 10 | 10 | 1,000 | 597 | -40.3 | -30.3 |
| 5 | 10 | 1,000 | 741 | -25.9 | -15.9 |
| 2 | 10 | 1,000 | 917 | -8.3 | +1.7 |
| 1 | 10 | 1,000 | 1,000 | 0.0 | +10.0 |

*Calculated with a financial calculator using Equation 3.

- Prices and returns for long-term bonds are more volatile than those for shorter-term bonds
- There is no interest-rate risk for any bond whose time to maturity matches the holding period


## The DiAtinction Between Real and Nominal Interest Rates

- Nominal interest rate makes no allowance for inflation
- Real interest rate is adjusted for changes in price level so it more accurately reflects the cost of borrowing
- Ex ante real interest rate is adjusted for expected changes in the price level
- Ex post real interest rate is adjusted for actual changes in the price level

Fisher Equation

$$
i=i_{r}+\pi^{e}
$$

$$
\begin{gathered}
i=\text { nominal interest rate } \\
i_{r}=\text { real interest rate } \\
\pi^{e}=\text { expected inflation rate }
\end{gathered}
$$

When the real interest rate is low, there are greater incentives to borrow and fewer incentives to lend.

The real interest rate is a better indicator of the incentives to borrow and lend.

## Figure 1 Real and Nominal Interest Rates (ThreeMonth Treasury Bill), 1953-2011



Quiz 1: calculate the M1 and M2 growth rate for 2010

|  | $\mathbf{2 0 0 9}$ | 2010 |
| :--- | :--- | :--- |
| Currency | 900 | 920 |
| Money Market Mutual Fund <br> shares | 680 | 681 |
| Saving account deposit | 5,500 | 5,780 |
| Money market deposit <br> accounts | 1,214 | 1,245 |
| Demand and checkable <br> deposits | 1,000 | 972 |
| Small denomination time <br> deposits | 830 | 861 |
| Traveler's check | 4 | 4 |
| 3-month treasury bill | 1,986 | 2,374 |


[^0]:    We need financial calculators to solve for the yield to maturity i.

